Homework 8, due Monday, July 25th

In this homework you will explore the method of undetermined coefficients to solve second order linear nonhomogenous differential equations.

An equation of the above type looks like $y'' + a_1 y' + a_0 y = g(x)$. To solve these we first have to get the **complementary solution**, usually denoted y_c, which is really just the general solution for the associated homogeneous version where we make the right side equal to zero. We figured these out in homework 7 by getting characteristic equations.

1) For each of the following differential equations, write what $g(x)$ is and the corresponding **complementary solution**.

a) $y'' + 4y' + 5y = 8x^2$

b) $3y'' + 6y' - 9y = e^{4x}$

c) $y'' + 4y = \sin(3x)$

Now to fully solve the differential equations above we need a **particular solution**. This can only be done when $g(x)$ is sufficiently nice, like a polynomial, an exponential, $\sin(x)$, or $\cos(x)$. This works by noticing that the left hand side is again just multiples of derivatives of the solution. And so if the right hand side looks like a polynomial of a certain degree then the left side must also and similarly for the other types.

We saw in class that this means we must guess a **particular solution** which looks like whatever is on the right, but in complete generality.

1) (cont.) For each differential equation write general form that the **particular solution** will have.

d) $y'' + 4y' + 5y = 8x^2$

e) $3y'' + 6y' - 9y = e^{4x}$

f) $y'' + 4y = \sin(3x)$

Finally to solve we plug the guess back into the differential equation and match coefficients to get the unknown constants giving us the particular solution. This process which gives a particular solution is called **the method of undetermined coefficients**. Then the final answer is the particular solution plus the complementary solution.

1) (cont.) Find the general solution $y = y_p + y_c$ to each differential equation using the method of undetermined coefficients to get each y_p.

g) $y'' + 4y' + 5y = 8x^2$

h) $3y'' + 6y' - 9y = e^{4x}$

i) $y'' + 4y = \sin(3x)$

We saw in class that this process can run into problems if the assumed particular solution has terms in common with the complementary solution. To fix this we just multpily the assumed particular solution by powers of x until there is nothing left in common!

2) For each differential equation write the general form that the **particular solution** will have.

a) $y'' + 4y' + 4y = 6e^{-2x}$

b) $y'' = 9x^2 + 2x - 1$

c) $y' - 5y = 2e^{5x}$

Now the fun part is figuring out what to assume solutions are when the function $g(x)$ is a product of linearly independent functions! It turns out to not be so difficult. Each function has its own type of assumed solution, and for each function we just multiply the assumed solutions together and adjust the constants. Then as usual multiply by powers of x if need be.

3) For each differential equation write the general form that the **particular solution** will have.

a) $y'' + 5y' + 6y = e^{3x}\cos(x)$

b) $y'' + 4y' + 4y = 4x^2e^{-2x}$

c) $y'' - y' - 2y = 3x^2\sin(5x)e^{3x}$

Finally, we can also deal with cases where $g(x)$ is a sum of functions in the previous cases. We just add the general forms together similarly to how we multiplied general forms together. This means we could deal with things as complicated as

$$y'' + 4y' - 9y = x^2e^{3x}\cos(2x) + 7e^{5x}\sin(x)x^4 + 2xe^{-2x}\sin(4x)$$

but of course this would take way too long so we won't.

4) Solve $y' - 5y = x^2e^x - xe^{5x}$.

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) Solve the following differential equations.

a) $y'' - y' - 2y = e^{3x}$

b) $y'' - y' - 2y = 5$

c) $y'' - y' - 2y = \cos(4x)$

d) $y'' - 6y' + 25y = 50x^3 - 36x^2 - 63x + 18$

e) $y' - 5y = (x-1)\sin(x) + (x+1)\cos(x)$ (The coefficients will be annoying fractions)

f) $y'' - 2y' + y = x^2 - 1$

g) $y'' - 2y' + y = 4\cos(x)$

h) $y'' - 2y' + y = xe^x$

i) $y' - y = xe^{2x} + 1$