Homework 7, due Tuesday July 19th

In this homework you will solve second order linear homogeneous differential equations with constant coefficients.

The long name of these types of equations sounds intimidating, but these are actually the easist equations to solve!

A **second order linear homogeneous differential equations with constant coefficients** looks like:

$$y'' + a_1 y' + a_0 y = 0.$$

This looks suspiciously similar to the following quadratic equation, which we call the **characteristic equation of the differential equation**:

$$x^2 + a_1 x + a_0 = 0.$$

Even more so when I write it like this:

$$x^2 + a_1 x^1 + a_0 x^0 = 0.$$

In fact there is a correspondence between the **zeroes** of the quadratic equation and the **solutions** of the differential equation. This correspondence was shown in class. Now to solve the differential equation all we have to do is solve the quadratic equation. This is some simple pre-calculus stuff! Just factor it or use the quadratic formula.

For historical reasons, the characteristic equation is usually written with a λ instead of an x, so we would say the characteristic equation is:

$$\lambda^2 + a_1 \lambda + a_0 = 0.$$

1) For each of the following differential equations, write the corresponding characterstic equation. (You may have to rewrite some of them first.)

a) $y'' + 4y' + 5y = 0$

b) $3y'' + 6y' - 9y = 0$

c) $y'' + 4y = 0$

d) $y'' + 2y' = 0$

e) $2y'' = 8y' - 16y$

Using the techniques learned in class do the following problems.

2) Solve $y'' - y' - 2y = 0$.

3) Solve $y'' - 5y = 0$.

4) Solve $y'' + 4y' + 5y = 0$.

5) Solve $y'' - 8y' + 16y = 0$.

The techniques used to solve these problems can be extended to higher order linear homogeneous differential equations. What I mean is if we have a differential equation that looks like:

$$y''' + a_2 y'' + a_1 y' + a_0 y = 0$$

It has a corresponding characteristic equation:

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0.$$

And finding the zeroes of this equation still corresponds to finding solutions of the differential equation. The only problem now is that factoring this third degree polynomial is a lot harder!

6) The differential equation $y''' - 6y'' + 11y' - 6y = 0$ has characteristic equation $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ which can be factored into $(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$. Using this, what is the general solution to the differential equation?

We can go to any order we'd like! The nth order differential equation:

$$y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = 0$$

corresponds to the characteristic equation

$$\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0 = 0$$

.

Again we would just factor this to get solutions, but factoring a very high degree polynomial by hand is very difficult and not worth our time so we won't worry about them. However if we're given some information we can solve higher order differential equations like in the following problem.

7) The differential equation:

$$y'''' - 15y'' + 10y' + 24y = 0$$

has two solutions $y = e^{2x}$ and $y = e^{3x}$. Use the correspodence between solutions and zeroes to find the general solution.

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) Solve the following differential equations.

a) $y'' - y = 0$

b) $y'' - 2y' + y = 0$

c) $y'' + y' + \frac{1}{4}y = 0$

d) $y'' + 25y' = 0$

e) $y'' + 25y = 0$

f) $y'' + 2y' + 3y = 0$

g) $y'' - y' - 30y = 0$