Homework 5, due Friday, July 8th

In this homework you will deal with the much dreaded **word problems**!

1) The following law of physics is known: **time rate of change of the temperature of an object is proportional to the temperature difference between the body and the surroundings**. Name the following quantities any variable you'd like.

a) Time =

b) Temperature of the object =

c) Temperature of the surroundings =

d) Constant of proportionality =

The idea is that we can use the above fact to figure out the temperature of some object. For example if I have a hot cup of coffee and leave it out in room temperature air, I can figure out how much cooler it will be at some time in the future. Since we're figuring out temperature of the object at various points in time, we will think of **the temperature of the object as a function of time** and the above fact tells us how to relate this function to it's derivative.

Namely, that **the derivative of the temperature with respect to time is equal to some constant times the difference between the function and surrounding temperature.**

2) Write down the differential equation using your variables which models the given law.

3) A metal bar at a temperature of 100°F is placed in a rooom at a constant temperature of 0°F.

a) Solve the differential equation from 2 with the room temperature of 0°F plugged in. (Note you should not be plugging the 100 in anywhere **yet**.)

This should give you a formula for temperature of the object, which looks something like $T(t) = ce^{-kt}$, but perhaps with different letters. If not fix your mistake or just use this formula from now on.

b) Since we know at time $t = 0$ that the temperature is 100°F we can plug in $t = 0$ and $T = 100$ to find c. Do this.

Now replace c in the formula to give you the specific formula for our specific problem with our specific initial conditions.

c) Now suppose that after 20 minutes the temperature of the bar is 50°F. Use this additional initial condition to find k.

d) Using this formula which has both our c and k plugged in, find the time it will take the bar to reach a temperature of 25°F.

e) Find the temperature of the bar after 10 minutes.

4) Using similar methods solve the following problem: A coffee cup at a temperature of 50°F is placed in a room at a constant temperature of 0°F. If after 10 minutes the temperature of the coffee is 25°F, find the time it will take the coffee to reach a temperature of 15 °F and the temperature of the coffee after 15 minutes.

5) Do the same problem but suppose the room is at a constant temperature of 10°F. This will change the initial differential equation slightly, but the rest of the steps will be the same.

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) For each word problem set up the differential equation which models it.

a) A theory of **epidemic spread** postulates that the growth of the number of zombies in a population of humans is proportional to the product of the number of zombies with the number of humans.

b) A certain radioactive material is known to decay at a rate proportional to the amount present.

c) As we all know, Spongebob Squarepants reproduces by **budding**. Scientists theorize that the growth of the Spongebob population is proportional to the number of Spongebobs, times the number of Patricks, minues the number of Squidwards.

P2) A body at an unknown temperature is placed in a room which is held at a constant temperature of 30°F. If after 10 minutes the temperature of the body is 0°F and after 20 minutes the temperature of the body is 15°F, find the unknown temperature. (Note that the differential equation used will be the same as in 2. We are just applying different initial conditions!)

P3) A body at a temperature of 50°F is placed outdoors where the temperature is 100°F. If after 5 minutes the body is 60°F, find how long it will take the body to reach a temperature of 75°F and the temperature of the body after 20 minutes. (Note that the differential equation used will be the same as in 2. We are just applying different initial conditions!)

P4) A radioactive material decays at a rate proportional to the amount present. As it decays it also cools at a rate proportional to the difference between the temperature of the material, 150°F initially, and the room temperature, 70°F. If initially there is 50 milligrams of the material present and after two hours it is observed that the material has lost 10% of its original mass and 20% of its temperature, find:

a) an expression for the mass of the material remaining at any time t,

b) the mass of the material after four hours,

c) the time at which the material has decayed to one half of its original mass,

d) an expression for the temperature of the material at any time t,

e) the temperature of the material after four hours,

f) the time at which the material has cooled to 80°F.