Homework 2, due Monday June 27th

1) Write the following differential equations in the form $y' + p(x)y = q(x)$ and then write what $p(x)$ and $q(x)$ are.

a) $y' = 10 - 15y$

b) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$

c) $xy' - 2y = x^5 \sin(2x) - x^3 + 4x^4$

2) Consider the differential equation given by:

$$\frac{1}{\sin(x)}y' = e^x y^3$$

a) Write this in the form $y' + p(x)y = q(x)y^n$. What is $p(x)$, $q(x)$, and n?

b) Write $y' = \frac{dy}{dx}$ and then write the differential equation as $f(x)dx = g(y)dy$. What is $f(x)$ and what is $g(y)$.

We've shown that this differential equation is Bernoulli and separable. This means that the classifications of differential equations have some overlap. Even better this means we will learn many ways to solve the same differential equations! So you don't have to do problems with a single technique.

c) Solve the differential equation. In other words find the function $y(x)$ which satisfies the differential equation.

3) Consider the differential equation

$$\frac{y'x^2 - y'y^2}{y} = 2x$$

a) Write this in the form $y' = f(x, y)$. What is $f(x, y)$?

b) Write out $f(tx, ty)$. Is it equal to $f(x, y)$?

c) Is the differential equation separable? Is it homogeneous? Is it linear? Is it Bernoulli?

d) Solve the differential equation.

4) **Show** that $y' = \dfrac{x+y+4}{x+y+2}$ is not homogeneous.

(Hint: When asked to **show something**, you should have some words in your answer. For example: We have that $y' = f(x,y)$ where $f(x,y) =$ _____. Then that means $f(tx,ty) =$ _____ and so that means _____ . Thus we conclude the differential equation is not homogeneous.)

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) Solve $y' = x^2 - 2x + 2$. (Hint: This requires you to remember how to do an integral which gives you arctan(x).)

P2) Solve $dy = 2t(y^2+9)dt$.

P3) Solve $y' = \dfrac{x^2+y^2}{xy}$ such that $y(1) = -2$.

P4) Solve $(t^2+1)dt + (y^2+y)dy = 0$.

P5) Solve $\dfrac{dy}{dx} = \dfrac{x+1}{y}$.

P6) Solve $y' = \dfrac{xe^x}{2y}$. (Hint: You'll need to remember integration by parts.)

P7) Solve $y' = \dfrac{2x^2+y^2}{xy}$.