Homework 1, due Wednesday June 22nd

1) Consider the differential equation:

$$\frac{dy}{dx} = x + 8$$

a) Check that $y(x) = \frac{x^2}{2} + 8x + 20$ is a solution to the differential equation.

b) Check that $y(x) = \frac{x^2}{2} + 8x + 85$ is also a solution.

c) More generally, check that $y(x) = \frac{x^2}{2} + 8x + \mathbf{C}$ is a solution where **C** is just some arbitrary constant.

You've shown that this differential has infinitely many solutions, one for every constant you can think of! This is very often the case and not at all like what happens when you solved equations back in algebra. If we want a specific solution we have to impose **initial conditions**.

d) If we need our solution to obey $y(0) = 8$, then the only solution is $y(x) = \frac{x^2}{2} + 8x + 8$. What is our only solution if we need it to obey $y(0) = 5$? What about y(2) = 16?

Just like in algebraic equations, some differential equations have **no solutions**. In real life applications most differential equations are very hard to solve! Keep this in mind as we go through techniques during the class.

2) Why does $(y')^4 + y^2 = -1$ have no solution? Your answer should feature a complete sentence that any English teacher would be proud of. (Hint: Think about positive and negative numbers.)

3) **Show** that $y(x) = 2e^{-x} + xe^{-x}$ is a solution of

$$y'' + 2y' + y = 0.$$

(Hint: When asked to **show something**, you should have some words in your answer. For example: If $y(x) =$ _______ then $y' =$ _______ and $y'' =$ _______ , so that if we plug them into the equation we get ________.)

Practice Problems - Do not turn these in

Doing the following problems will benefit you. Practice makes perfect and **math is not a spectator sport**.

P1) Check that $y(x) = \sin(x)$ is a solution of $\frac{dy}{dx} = \cos(x)$. Think about this and then see if you can figure out the solution to the differential equation $\frac{dy}{dx} = -\sin(x)$, what about $\frac{dy}{dx} = \sin(x) + \cos(x)$?

P2) Consider $y(x) = x^3 + 2x + 6$. Find $y(2), y'(3), y''(-1)$.

P3) Find the constants c_1, c_2 such that $y(x) = c_1 \sin(x) + c_2 \cos(x)$ satisfies:

a) $y(0) = 1, y'(0) = 2$

b) $y(\frac{\pi}{2}) = 1, y'(\frac{\pi}{2}) = 2$

c) $y(0) = 1, y'(\frac{\pi}{2}) = 1$

d) $y(0) = 1, y'(\pi) = 2$

P4) Find y', y'', $\frac{dy}{dx}$, y''' for the equation

$$y(x) = 2x^2 e^{3x} + 4x \sin(3x) - \frac{1}{\ln(2x)}$$